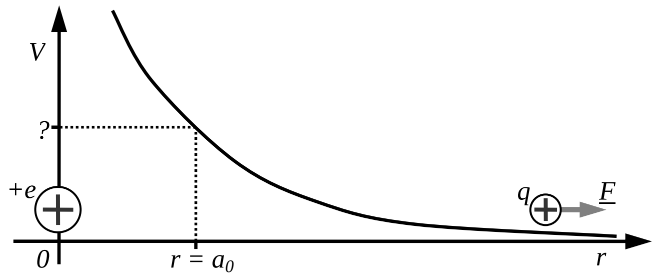
EXERCISE

Calculate the electric potential established by the nucleus of a hydrogen atom at the average distance (a_0 = 5.29x10^{-11} m) of the atom's electron (taking V = 0 at infinite distance).



The force F exerted upon a charge q by a charge $+e$ at a distance r is given by Coulomb's law

$$F = \frac{qe}{4\pi\varepsilon_0 r^2}$$

The potential energy of two charges is given by the work done to bring them together, where the work done against a force is equal to the force x distance moved against the force

$$\Delta E = E_2 - E_1 = F(-\Delta r)$$

The potential energy of our two charges, when separated by a_0, is therefore given by

$$E_{a_0} - E_\infty = -\sum_{r=\infty}^{r=a_0} F\,\Delta r$$

where the force F depends upon the separation r. We must therefore cast this as an integral,

$$E_{a_0} - E_\infty = -\int_\infty^{a_0} F\,\mathrm{d}r$$

which, inserting the particular form of the force from Coulomb's law, gives

$$\begin{aligned} E_{a_0} - E_\infty &= \int_\infty^{a_0} \frac{-qe}{4\pi\varepsilon_0 r^2}\,\mathrm{d}r \\ &= \frac{-qe}{4\pi\varepsilon_0}\int_\infty^{a_0} r^{-2}\,\mathrm{d}r \\ &= \frac{qe}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_\infty^{a_0} \\ &= \frac{qe}{4\pi\varepsilon_0}\left(\frac{1}{a_0} - \frac{1}{\infty}\right) \\ &= \frac{qe}{4\pi\varepsilon_0 a_0} \end{aligned}$$

The electric potential *V* is defined as the electrostatic potential energy per unit charge, ie

$$V = \frac{E}{q}$$

$\Rightarrow$

$$V_{a_0} - V_\infty = \frac{e}{4\pi\varepsilon_0 a_0}$$

and we may assume that *V* = 0 at *r* = ∞, so

$$V_\infty = 0$$

hence

$$V_{a_0} = \frac{e}{4\pi\varepsilon_0 a_0}$$

Given the specific values

e = $1.60\text{x}10^{-19}$ C
E_0 = $8.85\text{x}10^{-12}$ F.m^{-1}
a_0 = $5.29\text{x}1\text{—}^{11}$ m,

we obtain

$$V_{a_0} = \frac{1.6\times10^{-19}}{4\pi\times8.85\times10^{-12}\times5.29\times10^{-11}}\ \frac{\text{C}}{\text{F.m}^{-1}.\text{m}}$$
$$= 27.2\ \text{C.F}^{-1}$$

i.e.

$$V_{a_0} = 27.2\ \text{V}$$

As an example of the importance of identifying assumptions, we may take the following classic proof that $\log_{10}5$ is irrational.

We begin by assuming that $\log_{10}5$ is rational

i.e. we may write $$\log_{10} 5 = \frac{a}{b}$$

where *a* and *b* are integers.

$\Rightarrow$ $$5 = 10^{a/b}$$

$\Rightarrow$ $$5^b = 10^a$$

But any integer power of 5 must end in 5

$$5^b \equiv n\ldots nn5$$

and any integer power of 10 must end in 0

$$10^a \equiv 10\ldots 000$$

$\Rightarrow$ $$5 = 0$$

This is clearly false, so the initial (only) premise must be false,

i.e. $\log_{10}5$ is irrational.